SORITES WITHOUT VAGUENESS I: CLASSIFICATORY SORITES

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Abstract. An abstract mathematical theory is presented for a common variety of soritical arguments, treated here in terms of responses of a system, say, a biological organism, a gadget, or a set of normative linguistic rules, to stimuli. Any characteristic of the system's responses which supervenes on stimuli is called a stimulus effect upon the system. *Classificatory* sorites is about the identity of or difference between the effects of stimuli which differ 'only microscopically'. We formulate the classificatory sorites on arguably the highest possible level of generality and show that the 'paradox' is dissolved on grounds unrelated to vague predicates or other linguistic issues traditionally associated with it. If stimulus effects are properly defined (i.e., they are uniquely determined by stimuli), and if the space of the stimuli is endowed with appropriate (not necessarily metric) closeness and connectedness properties, then this space must contain points in every vicinity of which, 'however small', the stimulus effect is not constant. The effects can only be 'tolerant' to very small differences between stimuli if the closeness structure which is used to define very close stimuli does not render the space of stimuli appropriately connected: in this case the 'paradox' cannot be formulated. Nor can it be formulated if the response properties considered are not true effects, i.e., if they do not supervene on stimuli.

1. Introduction

We approach sorites as a behavioral issue, with 'behavior' broadly understood as the relationship between stimuli acting upon a system (which can be a biological organism, technical device, a set of rules, or anything whatever) and the system's responses to these stimuli. Examples of behavioral questions pertaining to sorites include: Can a person consistently respond by different characterizations, such as 'is 2 meters long' and 'is not 2 meters long', to visually presented line segments a and b which only differ by one billionth of one percent? Can a crude two-pan balance at equilibrium be upset by adding to one of the pans a single atom? Can the probability that this balance will remain at equilibrium change as a result of adding to one of the pans a single atom?

It is not essential for this approach whether the responses of the system are publicly observable. One can very well form behavioral soritical questions using as responses unobservable or hypothetical entities, such as molecular strains in a sheet of metal, perceptual images, or intentions. And even with observable responses, such as utterances of 'is long' and 'is not long', we may deal not with the

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responses per se (which in this example are not consistently determined by perceived physical length) but with some of their not directly observable properties, such as the probability with which one of these responses is evoked (which may be assumed to be consistently determined by physical length).[1] The essential feature of the behavioral approach is that a response or its property is only characterized by its occurrence or non-occurrence in conjunction with a stimulus and not by its meaning or truth value, even when these notions are applicable. It makes no difference whether one speaks of rusty balances tipping left or right in response to weights placed on their pans, flocks of birds migrating or staying in response to weather conditions, or 'competent users of language' assigning 'vague predicates' such as 'is long', 'is definitely beautiful', or 'is definitely definitely a fork' to perceived or described objects. Occurrences and non-occurrences of responses with certain properties are squarely within the realm of classical logic, and any contradiction derived from one's assumptions about stimuli and responses is a reductio ad absurdum proof that the conjunction of the assumptions is false. One can argue about the truth or falsity (or both, or neither) of stating that a line segment being presented is long, but the statement "in this trial this observer responded to this line segment by saying 'it is long' " is either true or false in the simplest sense, subject to no controversy.[2] Our goal is to show that statements of the latter kind form a sufficient basis for dissolving the 'sorites paradox', all semantic aspects of stimulus-response relations being irrelevant. As we argue in Section 2, normative judgments too, such as those formulated in terms of 'justifiability' of responses, can be recast in behavioral terms and thereby made to fall within the scope of our analysis. The soritical issues in our analysis have no special ties to the issue of vague predicates, however interesting in its own right. This is one reason for 'without vagueness' in the title of this paper.

The other reason lies in the fact that our analysis is formulated on the level of rigor of an axiomatic mathematical theory and arguably on the highest possible level of abstraction. This allows us to avoid superfluous considerations and the conceptual vagueness which underlay our naive intuitions of soritical issues and play their role in philosophical disagreements regarding them. In particular, we show that the notion of closeness of two stimuli which is required to describe soritical situations need not be quantitative or even topological: it may be defined by means of the much weaker concept of Maurice Fréchet's V-spaces.[3]

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[1] We will argue that consistent determination of the response properties by stimulus values is of critical importance: soritical arguments do not apply to inconsistent response properties (those whose values are not determined by the stimuli evoking them) or 'indeterminate' response properties (those whose values cannot be determined or at least assumed to be determinate).

[2] One might contend that in some cases response properties are in need of being measured or interpreted, and that precision limits of these measurements or uncertainties of the interpretations may render the response properties in question 'vague', or 'indeterminate'. This is not a serious problem for our analysis: the measurements or interpretations themselves simply take the place of the response properties in our analysis. If the measurements and interpretations are uniquely determined by the response properties being measured or interpreted, then the analysis presented in this paper fully applies. If these measurements and interpretations are inconsistent or indeterminate themselves, then, as any other inconsistent or indeterminate responses, they are not amenable to soritical arguments.

[3] Compare this with Timothy Williamson's (1994) position according to which closeness requires a full-fledged metric for its definition.

Sorites, viewed behaviorally, entails two different varieties of problems. The first, which we term *classificatory sorites* and address in this paper, is about the (non)identity of responses to stimuli which are 'almost identical', or differ 'only microscopically'. It is rooted in the original *sorites/phalakros* puzzles attributed to Eubulides: adding or removing one grain of sand or one hair at a time does not change whether the grains do or do not form a heap or the hairs a full head of hair. The second variety, *comparative sorites*, sometimes referred to in the literature as 'observational sorites', is about 'match/not match'-type responses to pairs of stimuli, and turns out to be very different from classificatory sorites. A treatment of comparative sorites is presented in a companion paper (Sorites Without Vagueness II).

This paper is organized as follows. An informal discussion of our treatment of the classificatory sorites, intended to provide a context and motivation for and to outline the applicable sphere of formal analysis, is given in Section 2 . The formal account itself follows in Section 3.

2. Informal Considerations

Consider a set S endowed with a *closeness structure*, so that for any $x \in S$ one can find in S an $x' \neq x$ which is 'as close as one wishes' to x (such as the set of real numbers with closeness defined by $|x - x'|$) or 'as close as possible' to x (such as the set of natural numbers with the same meaning of closeness). Let the elements of this set S represent *stimuli* acting upon some *system* and causing it to *react*. Thus, the stimuli may be electric currents passing through a digital ammeter which reacts by displaying a number on its indicator; or the stimuli may be schematic drawings of faces visually presented to a human observer who responds by saying that the face was 'nice' or 'not nice'; or the stimuli may be appropriately measured weather conditions in May to which a flock of birds reacts by either migrating or not migrate north. Almost any example would do, provided the closeness relation among stimuli is well-defined and responses have certain properties, such as the identity of the responses, the time it has taken to produce them, or their probability, that can be unambiguously attributed to stimuli. This means that everything else in the system's environment is held constant or is known to be irrelevant with regards to these properties.

2.1. **Supervenience, tolerance, and connectedness.** Consider now the following three characterizations of a hypothetical stimulus-response system.

Supervenience (Sup). Everything else being equal, the system cannot have different responses to different instances (repeated applications) of one and the same stimulus. That is, there is a function π such that the response of the system to stimulus x is $\pi(x)$. We call π the *stimulus-effect function*, and its values *stimulus effects*.

Tolerance (Tol). The stimulus-effect function $\pi(x)$ is 'tolerant to microscopic changes' in stimuli: if $x' \neq x$ is chosen sufficiently close to x, then $\pi(x') = \pi(x)$.

Connectedness (Con). The stimulus set S contains at least one pair of stimuli a, b with $\pi(a) \neq \pi(b)$ such that one can find a finite chain of stimuli

$$a = x_1, \ldots, x_i, x_{i+1}, \ldots, x_n = b,$$

leading from a to b 'by microscopic steps': x_{i+1} is arbitrarily or maximally close to but different from x_i for $i = 1, \ldots, n-1$.

The notions of closeness and connectedness by microscopic steps are, of course, yet to be made precise. But even at the present level of vagueness it is easy to see that the triple conjunction $\mathsf{Sup} \wedge \mathsf{Tol} \wedge \mathsf{Con}$ is self-contradictory, for it leads to the classical form of the *sorites 'paradox'*: if Sup and Con hold, then a pair of stimuli a, b with $\pi(a) \neq \pi(b)$ can be connected by a *soritical sequence* $x_1, \ldots, x_n$ where $a = x_1$, $b = x_n$, and every x_{i+1} is only 'microscopically' different from x_i; but then, by Tol, $\pi(x_i) = \pi(x_{i+1})$ for $i = 1, \ldots, n-1$, whence $\pi(a) = \pi(b)$. We call the obviously false hypothesis that the conjunction $\mathsf{Sup} \wedge \mathsf{Tol} \wedge \mathsf{Con}$ holds for some stimulus-response system the *classificatory* sorites.

It is easy to see that the conjunction $\mathsf{Sup} \wedge \mathsf{Tol} \wedge \mathsf{Con}$ is not only sufficient but also necessary for obtaining the soritical 'paradox'. Moreover, if Sup is not satisfied, i.e., stimulus effects are not determined by stimuli alone, then Tol and Con simply cannot be formulated as above, as these formulations make use of a stimulus-effect function $\pi(x)$. This invalidates the above reductio ad absurdum reasoning altogether.[4]

Once Sup is accepted, one can construct simple mathematical examples of consistent stimulus-response systems which do not satisfy at least one of Tol and Con. Thus, $\mathsf{Sup} \wedge \neg\mathsf{Tol} \wedge \mathsf{Con}$ is satisfied if S is the set of real numbers, closeness is defined by $|x - y|$, and π is any nonconstant function of the reals. With the same notion of closeness, the conjunction $\mathsf{Sup} \wedge \mathsf{Tol} \wedge \neg\mathsf{Con}$ is satisfied if S is the union of intervals

$$[0, 1], [2, 3], \ldots, [2n, 2n + 1], \ldots$$

and $\pi(x)$ is the greatest even integer not exceeding x (thus, e.g., $\pi(2.6) = \pi(3) = 2$). Finally, replacing $\pi(x)$ in the previous example with, say, $\pi(x) = x$, we get an example of a system satisfying $\mathsf{Sup} \wedge \neg\mathsf{Tol} \wedge \neg\mathsf{Con}$.

The only logical problem in every example of classificatory sorites is thus to find which of the three assumptions Sup, Tol, and Con is violated.

In most examples found in the philosophical literature the effect of a stimulus x, say, a line segment, is the assignment to x of a 'vague predicate' or its negation, such as 'is long' or 'is not long'. Whatever other properties we assign to vague predicates, however, the most salient one is that they are not assigned consistently, even when stimuli are known with complete physical precision. Charles S. Peirce, in his often cited dictionary article (1902), makes this inconsistency the defining property of vagueness. If any given length was consistently classified as 'long' or 'not long' by a given person or by all people in a group of 'competent speakers', the predicate 'is long' would not be considered vague for, respectively, this person or this group.[5] It follows that the assignment of vague predicates violates Sup, and

[4] One might wonder about the possibility of reformulating Con and Tol so that they could apply to inconsistent responses, those that are not stimulus-effect functions. With Con this can indeed be done, by requiring that *any* two points $a, b \in S$ be connectable by microscopic steps. Inconsistent responses, however, would immediately invaliadate Tol. Indeed, if one did not require that an effect of x be the same for all instances of x, then Tol would not be true even for $x' = x$, let alone for sufficiently close but different stimuli. Essentially the same argument precludes one from considering another way of relaxing Sup, the possibility that stimulus effects are determined by stimuli but their precise values cannot be known or even do not exist (however one understands this): then the equality of the effect of a stimulus x to that of a stimulus x' cannot be known either, or cannot even be meaningfully asserted.

[5] One can also consider as a system a set of normative rules prescribing in some manner the assignment of 'long' or 'not long' to every possible physical length: clearly, in this case, the assignment rules, if formulable at all, will satisfy Sup and therefore will have to violate at least one of the assumptions Tol and Con.

the relation between a set of stimuli and a set of vague predicates assignable to them is not a stimulus-effect function. One cannot therefore legitimately formulate a soritical 'paradox' using such assignments, and so faces one of two choices: either to seek additional factors which influence the choice between 'long' and 'not long' (in our example, besides physical length),[6] or to redefine the effect π in such a way that it becomes a function of length.

2.2. **Redefining stimulus effects to ensure supervenience.** The traditional approach in the behavioral sciences would be to redefine the stimulus-effect function $\pi = \pi(x)$ as the probability distribution of a random variable attaining values 'long' and 'not long': i.e., for some probability function $p(x)$,

$$\pi(x) = \begin{array}{|c|c|} \hline x \text{ is long} & x \text{ is not long} \\ \hline p(x) & 1 - p(x) \\ \hline \end{array},$$

or simply

$$\pi(x) = p(x) = \Pr[\text{`}x \text{ is long'}].$$

The function $p(x)$, called a *psychometric function* in psychophysics,[7] is well known to look more or less like the curve $p_2(x)$ shown in the middle panel of Fig. 2.1 (see, e.g., Luce 1963, Luce and Galanter 1963). We refer, of course, not to the exact shape of the curve but to the fact that the probabilities always change gradually and never like $p_1(x)$ in the left panel, as would be expected if the choice between 'long' and 'not long' was uniquely determined by physical length (under the assumption that $p(x)$ is nondecreasing).

To say that a probability p of a response to x is an effect of the stimulus x amounts to treating probabilities as occurring at individual instances of x 'within' the system responding to x, rather than characterizing patterns of the system's behaviors over a potential infinity of instances of x. This may cause philosophical concerns, which may even have some validity to them, although they are disregarded in the established conceptual schemes of probability theory, physics, and behavioral sciences. Thus, it is routine in the latter to speak of changes in the probability $p(x)$ from one instance of x to another as a result of learning or fatigue. The ontology of probabilities, however, is not a critical issue for our analysis. We only mention probability distributions as one possibility (a traditional one) of discussing the classificatory sorites in the face of inconsistent responses, such as utterances of vague adjectives, or, as far as human responses to perceptual stimuli are concerned, perfectly precise utterances—there seems to be no difference between the overall shapes of the psychometric functions for such responses as 'is long' and 'is longer than 2 meters'. If one denies that the probabilities of such inconsistent responses

[6] Thus, Diana Raffman's (1994, 1996) and Steward Shapiro's (2006) contextual approaches may be interpreted as asserting $\pi = \pi(x, b)$, where π attains values 'long' and 'not long', x is the physical length being judged, and b an unspecified spontaneous state (in the sense of being independent of x) of the observer's brain. A closeness structure should then be imposed on the set of (x, b)-values, e.g., by considering (x', b') close to (x, b) if x' is close to x in the usual physical sense and $b' = b$ (in which case it is easy to show that either Tol has to be violated or b alone should determine the effect π). The usefulness of this view depends on our ability to identify b in (x, b) without referring to the value of $\pi(x, b)$.

[7] Psychophysics, in the modern (and admittedly quite vague) sense of the word, is an area of psychology dealing with simple forms of comparative, classificatory, and evaluative judgments made in response to well-controlled stimuli. The adjective 'psychophysical' compared to 'psychological' implies a greater degree of conceptual and operational precision.

can be viewed as consistent stimulus effects, then one simply loses this possibility and has either to seek other 'hidden states' and/or 'hidden responses', or to declare sorites altogether unformulable (hence automatically dissolved).

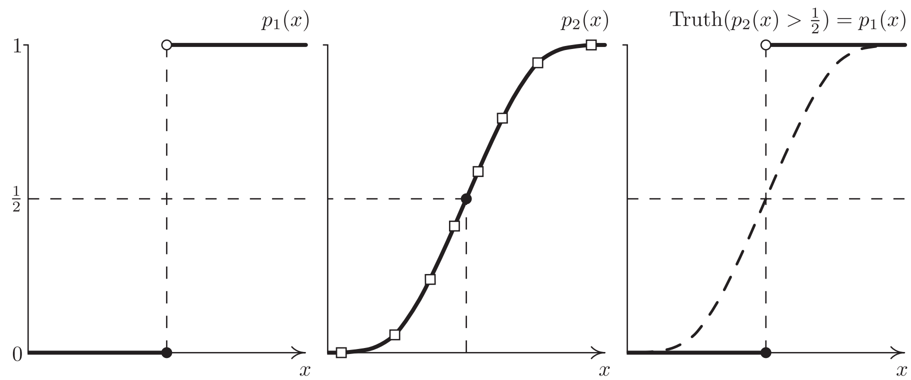

FIGURE 2.1. Examples of stimulus-effect functions for visually presented line segments x under the instruction to say whether a line segment is long (or longer than a standard segment, or longer than 2 m, etc.). Middle panel: a possible probability function $p_2(x)$ for saying 'long(er)', with squares representing values that could be estimated in a psychophysical experiment using eight randomly and repeatedly presented x-values. Left panel: a possible but unrealistic probability function $p_1(x)$, representing a deterministic choice between 'long(er)' and 'not long(er)'. Right panel: a deterministic stimulus-effect function obtained from the stimulus-effect function $p_2(x)$ by mapping its value to 1 if $p_2(x) > \frac{1}{2}$ and mapping it to 0 otherwise.

It has been proposed, primarily in the context of comparative sorites but applicable to classificatory sorites too, that the 'graduality' of a stimulus effect (say, a continuous mapping such as in the middle panel of Fig. 2.1) can be at least a significant part of a dissolution of the sorites 'paradox'.[8] We do not adopt this position here. It seems indeed to be the case that common intuition more readily allows for violations of Tol, the tolerance assumption, when the stimulus-effect function maps continuously into a set of reals than when it can only attain a finite number of values. Intuitions notwithstanding, however, the structure of the stimulus-effect function makes no difference for our analysis of sorites, as one can easily see from the fact that any function $\pi(x)$ can be transformed into a binary $\{0,1\}$-valued stimulus-effect function $\pi^*(x) = f(\pi(x))$ by partitioning the codomain of $\pi(x)$ into two nonempty disjoint sets and defining f as equal to 0 on one of them and to 1 on another. An example is shown in the right panel of Fig. 2.1. Variants of this

[8] See, e.g., Delia Graff (2001), but especially C.L. Hardin (1988) who specifically argues for a probabilistic understanding of human perceptual judgments (using one of the psychophysical signal detection models).

argument have been repeatedly used in the literature against what is known as the 'degree' approaches to sorites.[9]

2.3. **Redefining stimuli to ensure supervenience.** In an attempt to properly construct a stimulus-effect function π one may also look into various ways of (re)defining the stimuli, though in a less dramatic fashion than the one mentioned in footnote 6. Thus, one might think it important to take into account sequential effects, the dependence of a stimulus effect on a sequence of previously presented stimuli, or even on both the previous stimuli and the responses given to them. For instance, given a stimulus set I, one can assume that the stimulus-effect function is determined by compound stimuli

$$x^* = (x_{-k}, \ldots, x_{-1}, x_0), \qquad k \geq 1,$$

where $x_0 \in I$ is the stimulus currently presented, and $x_{-i-1} \in I$ is the stimulus presented prior to stimulus $x_{-i} \in I$ $(i = 0, \ldots, k-1)$. These compound stimuli will now form the 'true', 'properly defined' stimulus set S. (The definition should be modified in the obvious way if x_0 is preceded by less than k stimuli in I since the beginning of the count.) If the initial stimulus set I is endowed with some closeness structure, call it closeness 'in the initial sense', then two compound stimuli $x^* = (x_{-k}, \ldots, x_{-1}, x_0)$ and $y^* = (y_{-k}, \ldots, y_{-1}, y_0)$ can be naturally defined as being close if y_0 is close to x_0 in the initial sense, and y^* extends x^* in the sense that $y_{-i} = x_{-i+1}$ for $i = 1, \ldots, k$. This definition, and many similar variants, would ensure that for any sequence of stimuli in I each of which is close to its predecessor, the same is true for the corresponding sequence of compound stimuli in S: e.g., putting $k = 3$ for concreteness, if in the sequence

$$x_0, x_1, x_2, \ldots$$

each x_{i+1} differs from x_i 'microscopically', then the same is true for the sequence $x_0^*, x_1^*, x_2^*, \ldots$, where $x_0^* = (x_0), x_1^* = (x_0, x_1)$, and for each $i \geq 2$,

$$x_i^* = (x_{i-2}, x_{i-1}, x_i).$$

If now a stimulus-effect function π, such as the probability of saying 'long', is uniquely determined by such a triad of successive stimuli, the characterizations Sup, Tol, and Con above will apply with no modifications, and so will the formal analysis presented in Section 3.

If the stimulus-effect function π is assumed to be determined not only by previously presented stimuli in I but also by the (possibly inconsistent) responses they evoked, the stimuli comprising the 'properly defined' set S can be defined as

$$x^{**} = \begin{array}{|c|c|c|c|c|}\hline x_{-k} & x_{-k+1} & \ldots & x_{-1} & x_0 \\ \hline r_{-k} & r_{-k+1} & \ldots & r_{-1} & \\ \hline \end{array}\,,$$

where r_{-i} for $i = 1, \ldots, k$ is the response associated or co-occurring in the same trial with $x_{-i} \in I$. Note that the stimulus-effect function $\pi(x^{**})$ and the generally inconsistent responses r in this example may but need not belong to the same set.

[9]See, e.g., James Cargile (1969). The argument can also be used to manipulate one's intuitions to reject the relevance of the 'graduality' of stimulus effects to classificatory sorites, although it is usually equally effective to simply ask questions like "Do you really think that the probability with which you can say 'heavy' may change if one adds a single atom to the weight being judged?"

A typical example of r_{-i} would be the words 'long' or 'not long'. The stimulus-effect function $\pi(x^{**})$ may be one of these words too, but it could also be, e.g., the probability distribution

$$\pi(x^{**}) = \begin{array}{|c|c|} \hline x_0 \text{ in } x^{**} \text{ is long} & x_0 \text{ in } x^{**} \text{ is not long} \\ \hline p(x^{**}) & 1 - p(x^{**}) \\ \hline \end{array}.$$

With an 'initial' closeness structure imposed on I, the compound stimulus x^{**} will be considered close to

$$y^{**} = \begin{array}{|c|c|c|c|c|} \hline y_{-k} & y_{-k+1} & \cdots & y_{-1} & y_0 \\ \hline r'_{-k} & r'_{-k+1} & \cdots & r'_{-1} & \\ \cline{1-4} \end{array}$$

if y_0 is close to x_0 'in the initial sense', and $(y_{-i}, r'_{-i}) = (x_{-i+1}, r_{-i+1})$ for each $i = 1, \ldots, k$. Again, our analysis will include thus defined stimulus-response relations as a special case, with no modifications or additional considerations required. This would apply to other models of sequential effects, e.g., the dependence of π on all stimuli preceding the given one, or on a randomly determined number of preceding stimuli. One can also entertain, with no modifications to the general analysis, other closeness structures induced among compound stimuli in S by the 'initial' closeness among elements of I.

We must also mention what is perhaps the most radical way to satisfy the supervenience requirement Sup: by including stimulus instances in the stimulus identities. This means that each stimulus is formally characterized by a pair (x, t) where x belongs to an initial set I of physical values, and t designates an 'instance', defined by a time interval or trial number.[10] Let it be the latter. A response to (x, t), whatever this response may be, is a stimulus effect due to the fact that the stimulus (x, t) can never be replicated. Assuming again some 'initial' notion of closeness among elements of I, a natural way to define closeness between (x, t) and (y, t') is to require that y be close to x in the initial sense and that t' be the trial following t, i.e., $t' = t + 1$.[11] The requirements Tol and Con for the newly defined stimulus set S are formulated as follows:

Special case of Tol. If $(x', t + 1)$ is chosen sufficiently close to (x, t), then the responses in the trials t and $t + 1$ are identical.

Special case of Con. S contains at least one pair of stimuli $(a, t), (b, t + n)$, $n > 0$, with different responses in the trials t and $t + n$, such that one can find a chain of stimuli

$$(a, t) = (x_1, t), \ldots, (x_i, t + i - 1), (x_{i+1}, t + i), \ldots, (x_{n+1}, t + n) = (b, t + n),$$

where $(x_{i+1}, t+i)$ is arbitrarily or maximally close to $(x_i, t+i-1)$ for $i = 1, \ldots, n$.

In spite of the very peculiar definition of stimuli in this example, the incompatibility of Tol and Con in this setting, with Sup being satisfied 'automatically', is obvious. Therefore the formal analysis presented in Section 3 includes this radical approach as a special case calling for no special considerations.

[10] We by no means suggest that this approach is reasonable, only that it also falls within the scope of our formal theory.

[11] This is in fact how sorites is often described (a repeated question about a series of gradually changing stimuli), sometimes referred to as the 'forced march sorites' (see, e.g., Shapiro, 2006).

2.4. **No-tolerance 'paradox'.** Assuming now that the supervenience requirement, Sup, is satisfied by an appropriate choice of stimuli and response properties, and assuming in addition that the closeness structure is chosen so that Con is satisfied, we face a simple conclusion: $\mathsf{Sup} \wedge \mathsf{Con} \Longrightarrow \neg\mathsf{Tol}$, where for emphasis we recall that $\neg\mathsf{Tol}$ is the statement

Non-tolerance ($\neg\mathsf{Tol}$). There is at least one point $x_0 \in S$ in every vicinity of which, however small, the stimulus-effect function $\pi(x)$ is nonconstant.

In fact, in presenting classificatory sorites, Sup and Con are almost always assumed implicitly, although Sup is sometimes mentioned as an innocuous premise. The 'paradox' then consists in pointing out that people often find hard to accept $\neg\mathsf{Tol}$, and that those who are willing to accept it can be further confused by being asked to point out the location of such an x_0 in the stimulus set S.

We do not view confusions in people's theorizing about their possible behaviors as paradoxes. People are known to maintain wrong beliefs about many subjects, from trajectories of bodies moving by inertia to other people's motives. Such beliefs, however, are hardly relevant to an objective analysis of inertia or people's motives. With regards to soritical intuitions, we agree with Achille C. Varzi (2003) who points out that they fall outside the sphere of a logical or philosophical analysis.[12] The implication $\mathsf{Sup} \wedge \mathsf{Con} \Longrightarrow \neg\mathsf{Tol}$ is a theorem, stated rigorously and in complete generality in Section 3, hence $\neg\mathsf{Tol}$ must be true in the behavior of any real system satisfying Sup and Con; and if one holds that such systems do not exist, then one should also accept that the 'sorites paradox' cannot even be formulated.[13] In this respect, the 'epistemic' dissolution of the classificatory sorites proposed by Roy A. Sorensen (1988a-b) and Timothy Williamson (1994, 2000) is correct: a point or points x_0 with the property stipulated in $\neg\mathsf{Tol}$ must exist objectively for any system satisfying Sup and Con. In fact, it may very well be that every single point in S has this property, as it is the case in the middle panel of Fig. 2.1 and in a host of other situations with continuously varying stimulus effect.

Contrary to the epistemic approach, however, such points x_0 can be found and identified to any degree of precision, provided the system in question maintains its identity for the duration it is studied. Thus, the psychometric function $p_2(x)$ in Fig. 2.1 and the points where it reaches a particular level, such as $\frac{1}{2}$ in the right panel of the figure, are routinely estimated in psychophysical experiments. The participants in such an experiment, of course, will most likely have no idea where the median or any other feature of their psychometric functions might be, or even whether their responses to a given stimulus are deterministic or probabilistic. We also disagree with the epistemicists when they assert a 'sharp boundary' between stimuli characterized by a vague predicate, such as 'long', and those characterized by the predicate's negation: such a boundary would have existed only if vague predicates were assigned to stimuli consistently, which they are not by virtue of the very fact that they are vague. In this regard the 'degree' theorists and supervaluationists are more in the right, the former by replacing the dichotomies of the 'long/not long' type by gradual effects which can more plausibly be assumed to supervene on stimuli (e.g., Edgington 1999), the latter by emphasizing the essential arbitrariness

[12]Varzi also comes closer than many to the 'behavioral' approach by pointing out that soritical issues are essentially non-semantic and are not confined to linguistic phenomena.

[13]Such systems, however, can be easily constructed: consider a set of length values, for instance, with the conventional closeness structure and connectedness property, and put $\pi(x) = x$

with which 'normative' truth values can be assigned to vague predicates paired with stimuli (Fine 1975). Both these approaches, however, as well as the epistemic one, differ from ours in their treating soritical issues in terms of truth values of linguistic constructs.

It clearly makes no difference for our approach whether we deal with predicates like 'is long' or like 'is definitely long', 'is definitely definitely long', etc. In all cases the predicate is treated as a response, and it seems very likely that no such a response supervenes on stimuli if the latter are objects of different length visually presented or described. And if one of these predicates, or, more likely, the probability of invoking it, did supervene on stimuli (endowed with the usual closeness structure), the no-tolerance conclusion would follow. There is therefore no special place or significance for considerations of 'higher-order vagueness' in the behavioral approach.

2.5. **Normative considerations.** One might accept our analysis as applied to systems responding to stimuli, but still argue that it leaves out a class of soritical considerations which pertain to the normative category of 'justification' for responses of a system (then necessarily a sentient one). A human responder may, this argument goes, change his or her factual responses in every vicinity of a particular stimulus value in accordance with $\neg\mathsf{Tol}$, but will be unable to justify these responses, to explain why this particular response to this stimulus ought to be chosen over other, competing responses. This argument is untenable. Such judgments as "this response is (non)arbitrary", or "this choice is (un)justified" can always themselves be viewed as *responses* to appropriately defined stimuli. We are dealing then with one of two situations: either with possible distributions of these special responses over a set of stimuli, in which case our analysis applies with no modifications, or with people's (often erroneous) theorizing about these behaviors, which is a topic best left to psychologists interested in 'naive' conceptual schemata.

Consider, e.g., the problem of setting a minimum height requirement for children riding a roller coaster. The roller coaster operator who decides to set this minimum height at x cm can be asked various questions related to 'justifiability' of this choice. Thus, the operator is likely to concede that the choice of x cm will be unfair for a child whose height is $x-\varepsilon$ cm provided ε is sufficiently small. One might argue that this can be said for any x proposed as the boundary, whence a soritical paradox ensues because the operator definitely knows that a child under, say, 50 cm of height must not ride the roller coaster. This paradox, however, is dissolved by recasting the situation in terms of a system (the roller coaster operator) acted upon by stimuli (various choices of x) and responding by saying "this value can/cannot be lowered" or "I am/am not absolutely sure this value can be lowered". If the choice of allowable responses and the questioning procedure are such that the responses are consistent, so that Sup is satisfied and the paradox can even be formulated, then, given that the closeness and connectedness properties here are ensured, a value x_0 must exist, say ≥ 50 cm, to which the roller coaster operator will respond "this value cannot be lowered" (or "I am not absolutely sure this value can be lowered"). One could try to salvage the paradox by defining a choice of $x = x_0$ as 'justifiable' if the 'reasons' for this choice, whatever they may be, do not apply to any other value of x. The operator can then be asked, in response to various values of x, whether this value is justifiable in this sense. On a moment's reflection, however, only two outcomes are possible in this situation, both contingent on the assumption that the operator responds consistently: either the operator will think of reasons for picking a particular precise value, which would accord with $\neg\mathsf{Tol}$, or not. In the latter case, the paradox does not obtain simply because the operator's response does not vary: every x is deemed 'unjustifiable' in the sense that it could very well be either slightly lowered or slightly incremented (or both), on the entire set of possible height values.

2.6. **No-tolerance does not lead to absolutely precise measurements.** We should briefly address the question which may be raised in connection with $\neg\mathsf{Tol}$: if this is the case, why can't one use the responding system in question to measure some of the stimulus values with absolute precision? The answer seems to be more subtle than complex. In order to distinguish a stimulus x from its arbitrarily close neighbors x' by means of a stimulus-effect function $\pi(x)$ a human researcher should possess a system identifying stimuli being presented, $\iota_S(x)$, and a system identifying the stimulus effects being recorded, $\iota_R(\pi(x))$. The former is needed to ensure that the researcher knows that x being presented on two different instances is indeed one and the same x, and that x' presented on another instance is not the same as x. Otherwise, if it is not known which of the two stimuli is presented, x or x', and $\pi(x) \neq \pi(x')$, the researcher may have to conclude that $\pi(x)$, assuming for now it is known precisely, is ill-defined as a stimulus-effect function. It will then have to be redefined, e.g., as an x-dependent probability distribution over the values of $\pi(x)$. But to have such an identification function $\iota_S(x)$ amounts to having yet another stimulus-effect function, besides $\pi(x)$, whose values react to arbitrarily small differences from precisely the same stimulus x—something not impossible but definitely not deliberately construable (unless the stimuli have been identified by some $\iota'_S(x)$, which assumption would lead to an infinite regress).

Turning now to the identification $\iota_R(\pi(x))$ of responses, it is trivial only if the values of $\pi(x)$, at least in the vicinity of the x in question, are discrete, i.e., $\pi(x)$ 'jumps' by at least some minimal fixed amount as x changes to x', however close to x. Otherwise, if $\pi(x)$ represents degrees or probabilities continuously changing, say, on an interval of reals, the problem of knowing $\pi(x)$ becomes as formidable as that of knowing x, and perhaps more so if $\pi(x)$ is a principally unobservable quantity, such as probability. This explanation seems close to Williamson's (1994) 'margin of error' conception.

3. A Formal Treatment of Classificatory Sorites

Our informal discussion shows that to clearly formulate the classificatory sorites one needs three things:

(1) A set of *stimuli* S endowed with a *closeness structure*. The latter should allow one, for every pair of stimuli $x, x' \in S$, to characterize 'how close' x' is to x. Thus, S may be a set of horizontal line segments, and the conventional distance $|x - x'|$ between two lengths may be used to characterize closeness of x' to x. As explained below, however, the 'closeness' does not have to be a numerical measure: it is generally defined in qualitative (in fact, even more basic than topological) terms.

(2) A '*connectedness*' structure on S which allows one, for at least some pairs of elements $a, b \in S$, to form finite chains of stimuli

$$a = x_1, x_2, \ldots, x_{n-1}, x_n = b$$

'connecting a to b' with successive elements 'as close as one wishes', or 'as close as possible'. Thus, if the length values in our example of S form an interval of positive reals, any two line segments can be connected by a chain of line segments whose lengths are spaced arbitrarily densely, in the $|x_{i+1} - x_i|$ sense, within this interval.

(3) A *stimulus-effect function* $\pi : S \to R$ which assigns to each element of S a unique element of some set R. The elements of R (*stimulus effects*) may be observed or computed from observed responses.

3.1. **V-spaces of stimuli.** The notion of closeness is formalized by means of V-spaces introduced by Maurice Fréchet (1918), a relatively little known concept which is arguably the most general possible way of speaking of closeness.

Definition 3.1. Given a nonempty set S, a *V-space* on S is a pair

$$\{S, \{\mathcal{V}_x\}_{x\in S}\}$$

where $\mathcal{V}_x$, for each $x \in S$, is a collection of subsets of S satisfying

(1) $\mathcal{V}_x \neq \varnothing$,
(2) if $V \in \mathcal{V}_x$ then $x \in V$.

For each $x \in S$, any element V of $\mathcal{V}_x$ is called a *vicinity* of x.[14] Any set of vicinities obtained by choosing one element of $\mathcal{V}_x$ for every $x \in S$ is called a *V-cover* of S.

Intuitively, $V \in \mathcal{V}_x$ is a set of stimuli which are close to x 'in some sense' (as a special case, 'to some degree'). The set

$$\mathcal{C}_x^{x'} = \{V \in \mathcal{V}_x : x' \in V\}$$

of the vicinities of x that contain x' characterizes 'how close' x' is to x (i.e., in 'what senses' or to 'what degrees' it is close to x). In particular, x is close to itself in all possible (for x) senses:

$$\mathcal{C}_x^x = \mathcal{V}_x.$$

See Fig. 3.1 for a schematic illustration.

If S is endowed with a metric d, the set $\mathcal{V}_x$ can be chosen as the set of the open balls

$$B_x(\varepsilon) = \{u \in S : d(x,u) < \varepsilon\}$$

for all $\varepsilon > 0$, and $\mathcal{C}_x^{x'}$ is then uniquely determined by $d(x,x')$:

$$\mathcal{C}_x^{x'} = \{B_x(\varepsilon) : d(x,x') < \varepsilon\}.$$

In general, however, the closeness of an x' to x does not have to have a numerical sense. Thus, if S is a topological space, the set $\mathcal{V}_x$ of the vicinities of x may be chosen to coincide with the set of all (open) neighborhoods of x, and $\mathcal{C}_x^{x'}$ then consists of the neighborhoods of x which contain x'.[15]

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[14]We translate Fréchet's 'voisinage' as 'vicinity', because the term 'neighborhood' is firmly associated with topological spaces (a special case of V-spaces). In fact, Fréchet's definition does not even require that $x \in V$ for $V \in \mathcal{V}_x$. To require this, however, is more intuitive and does not diminish the generality of the construction (see Sierpinski 1956, p. 5). It may seem very intuitive to posit also that $V \cap U \in \mathcal{V}_x$ for any $V, U \in \mathcal{V}_x$. This would create structures stronger than V-spaces but weaker than topological spaces. We do not, however, need this in the present context.

[15]The set of all vicinities $\{\mathcal{V}_x\}_{x\in S}$ becomes a topological basis if we postulate that, whenever $z \in U \cap V$ (where $U \in \mathcal{V}_x$ and $V \in \mathcal{V}_y$), there is a $W \in \mathcal{V}_z$ such that $W \subset U \cap V$. The corresponding topology on S (i.e., the set of open subsets of S) then is obtained as the set of the

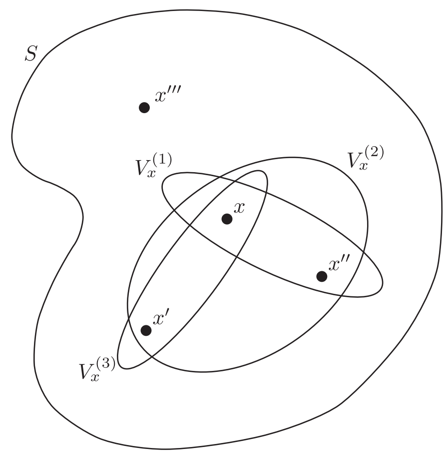

FIGURE 3.1. An example of a V-space $\{S, \{\mathcal{V}_x\}_{x \in S}\}$. S consists of all points within the large outlined area. A point $x \in S$ is shown with its set of vicinities $\mathcal{V}_x = \{V_x^{(1)}, V_x^{(2)}, V_x^{(3)}\}$. The point x' is close to x in the sense $\mathcal{C}_x^{x'} = \{V_x^{(2)}, V_x^{(3)}\}$, x'' in the sense $\mathcal{C}_x^{x''} = \{V_x^{(1)}, V_x^{(2)}\}$, and x''' in no sense (is not close at all), as $\mathcal{C}_x^{x'''} = \varnothing$.

3.2. **V-connectedness.**

Definition 3.2. A point $a \in S$ is *V-connected to* a point $b \in S$ in a V-space $\{S, \{\mathcal{V}_x\}_{x \in S}\}$ if for any V-cover $\{V_x\}_{x \in S}$ of S one can find a finite chain of points $x_1, x_2, \ldots x_{n-1}, x_n \in S$ such that

(1) $a = x_1$,
(2) $b = x_n$,
(3) $V_{x_i} \cap V_{x_{i+1}} \neq \varnothing$ for $i = 1, \ldots n-1$.

A V-space $\{S, \{\mathcal{V}_x\}_{x \in S}\}$ is *V-connected* if any two points in S are V-connected in $\{S, \{\mathcal{V}_x\}_{x \in S}\}$.[16]

See Fig. 3.2 for a toy example. For a more interesting example, let S be a set of natural numbers $0, 1, \ldots$, and let $\mathcal{V}_n$ consist of all sets $\{n, n+1, \ldots, n+k\}$ with $0 < k \leq K_n$, where K_n is some nonvanishing function of n (Fig. 3.3 provides an example with $K_n \equiv 1$). A space of this structure is needed, e.g., for formulating the classical phalakros and the original sorites paradoxes, where the respective amounts of hair and grains are natural numbers. Any two numbers m and n

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unions of all possible collections of vicinities. Note that unlike in the general case, the topological notion of closeness is symmetric: $\mathcal{C}_x^{x'} = \mathcal{C}_{x'}^{x}$.

[16]V-connectedness is not a standard generalization of topological connectedness to V-spaces. The standard one (Sierpinski 1956, Chapter 1) is to consider a V-space $\{S, \{\mathcal{V}_x\}_{x \in S}\}$ connected if S does not have a proper open subset A such that $S - A$ is open too (where we define an open set as one whose every element has a vicinity contained in the set). Let us call such a space S-connected. The two notions coincide if the space is topological, but in general V-connectedness implies S-connectedness without being implied by it. If the space is not S-connected, then for any decomposition of it into nonempty open A and $S - A$, any two V-connected points of S either both belong to A or both belong to $S - A$.

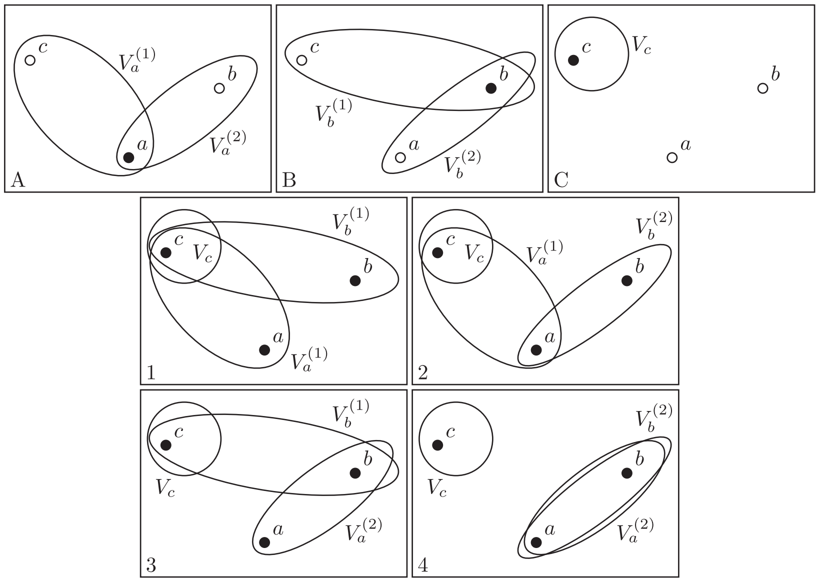

FIGURE 3.2. A V-space $\{S, \{\mathcal{V}_x\}_{x\in S}\}$ with the three-point set $S = \{a, b, c\}$. Panels A, B, and C show the sets of vicinities $\mathcal{V}_a$, $\mathcal{V}_b$, and $\mathcal{V}_c$, respectively. Thus, $\mathcal{V}_a$ consists of $V_a^{(1)} = \{a, c\}$ and $V_a^{(2)} = \{a, b\}$ (unlike in Fig. 3.1 the vicinities contain no points besides a, b, c); $\mathcal{V}_b$ consists of $V_b^{(1)} = \{b, c\}$ and $V_b^{(2)} = \{a, b\}$; $\mathcal{V}_c$ consists of the single singleton vicinity $V_c = \{c\}$. The four panels $1, 2, 3, 4$ show all possible V-covers of S: e.g., the V-cover shown in panel 1 is obtained by choosing $V_a^{(1)}$ from $\mathcal{V}_a$, $V_b^{(1)}$ from $\mathcal{V}_b$, and V_c from $\mathcal{V}_c$. It is easy to see that a is V-connected to b: for the V-cover $\{V_a^{(1)}, V_b^{(1)}, V_c\}$ in panel 1 (a, c, b) is a sequence connecting a to b; for the remaining three V-covers such a sequence is (a, b). Panel 4 shows that neither a nor b is connected to c. In reference to Lemma 3.3, $\{a, b\}$ and $\{c\}$ are the two V-components of the space.

in this space are V-connected because for any choice of $k_0, k_1, \ldots$ the vicinities $\{m, \ldots, m+k_m\}, \{m+1, \ldots, m+1+k_{m+1}\}, \ldots, \{n, \ldots, n+k_n\}$ satisfy the properties (1)-(3) of Definition 3.2. If, however, k_n is allowed to vanish for some n, some pairs of natural numbers may not be V-connected (e.g., if k_0 can be 0, then $n = 0$ will not be V-connected to any $m > 0$).

Another example: if S is the set of rational numbers, no two elements of S are V-connected if the vicinities are defined in the common, topological, way: $\mathcal{V}_r$ for $r \in S$ consists of all intervals $]r - q, r + q[\cap S$, where q is a positive rational number; but the space is V-connected if q in the definition of $\mathcal{V}_r$ is not allowed to fall below some $\varepsilon > 0$.

It is easy to see that the relation of 'being connected to' is an equivalence relation, whence we immediately have the following lemma.

Lemma 3.3. *For any V-space $\{S, \{\mathcal{V}_x\}_{x\in S}\}$, the set S is a union $\bigcup S_\gamma$ of pairwise disjoint nonempty subsets (the* V-components *of S) such that any two points in*

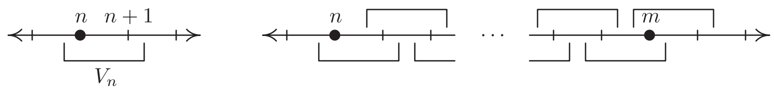

FIGURE 3.3. A V-space which is sufficient to formulate sorites for integer-valued stimulus spaces: each integer n has a single vicinity $\{n, n+1\}$, and any two points n and m are V-connected.

every V-component are V-connected in $\{S, \{\mathcal{V}_x\}_{x\in S}\}$ *and no two points belonging to different V-components are V-connected in* $\{S, \{\mathcal{V}_x\}_{x\in S}\}$.[17]

3.3. **Stimulus-effect function.**

Definition 3.4. Given a V-space $\{S, \{\mathcal{V}_x\}_{x\in S}\}$ and an arbitrary set R, any function $\pi : S \to R$ is a *stimulus-effect function* (and R a set of *stimulus effects*). A stimulus-effect function π is called *tolerant at* $x \in S$ in $\{S, \{\mathcal{V}_x\}_{x\in S}\}$ if there is a vicinity $V_x \in \mathcal{V}_x$ on which π is constant; π is *tolerant* if it is tolerant at every point. [18]

The variety of ways in which one can define R has been discussed in the previous section. The following are two obvious properties of stimulus-effect functions:

(1) If $\pi : S \to R$ is a stimulus-effect function, then so is $f \circ \pi : S \to R^*$ where f is any function $R \to R^*$.
(2) If $\{\pi_\upsilon : S \to R_\upsilon\}_{\upsilon\in\Upsilon}$ is a collection of stimulus-effect functions (with Υ an arbitrary indexing set), then $\pi : S \to \prod_{\upsilon\in\Upsilon} R_\upsilon$ is a stimulus-effect function too, where $\prod$ stands for the Cartesian product and $\pi(x) = \{\pi_\upsilon(x)\}_{\upsilon\in\Upsilon}$.

The first property allows one to 'coarsen' a given stimulus-effect function in any desirable way, e.g., to create a binary function from a multivalued one (as in Fig. 2.1). The second property allows one to combine different stimulus-effect functions (e.g., elicited under different instructions in a psychophysical experiment) into a single one. The conjunction of the two properties allows one to say that given a stimulus-effect function π, the system can always be viewed as possessing the stimulus-effect function $\pi^*(x) = \{f_\upsilon \circ \pi(x)\}_{\upsilon\in\Upsilon}$ for any set of functions $\{f_\upsilon\}_{\upsilon\in\Upsilon}$ defined on R.

3.4. **No-tolerance theorem.** This is the main theorem of this section (together with Theorem 3.10 below which represents an alternative approach).

Theorem 3.5. *Let* $\{S, \{\mathcal{V}_x\}_{x\in S}\}$ *be a V-space and* $\pi : S \to R$ *a stimulus-effect function, such that* S *contains two V-connected elements* a, b *for which* $\pi(a) \neq \pi(b)$. *Then* π *is not tolerant: there is at least one* $x \in S$ *such that* π *is nonconstant on any vicinity of* x *('however small').*

Proof. Assume π is tolerant: every x has a vicinity V_x^* such that π is constant on V_x^*. The set $\{V_x^*\}_{x\in S}$ is a V-cover of S, and a, b being V-connected, one can form a sequence $V_{x_1}^*, \ldots, V_{x_n}^*$ satisfying (i)-(iii) of Definition 3.2. Then, denoting by y_i

[17] In accordance with footnote 16, if S can be decomposed into nonempty open A and $S - A$, then every V-component is entirely contained in A or entirely contained in $S - A$.

[18] The term 'tolerant' is chosen because it is used in the philosophical literature related to sorites. Following the term-formation scheme adopted in this paper it would have been more logical to label tolerant functions 'V-locally constant' ('locally constant function' is a standard mathematical term for the analogous property in topological spaces).

an arbitrary element of $V^*_{x_i} \cap V^*_{x_{i+1}}$, we would have $\pi(y_i) = \pi(y_{i+1})$ (since $y_i, y_{i+1} \in V^*_{x_{i+1}}$, $i = 1, \dots n-1$), whence $\pi(a) = \pi(x_1) = \pi(x_n) = \pi(b)$, contradicting the premise $\pi(a) \neq \pi(b)$. $\square$

One can easily see in the proof of this theorem the spelled-out version of the classical (classificatory) sorites, taken as a reductio ad absurdum proof of the incompatibility of Sup, Tol, and Con, informally stated in the previous section.

Corollary 3.6. *No nonconstant function on a V-connected V-space is tolerant.*

3.5. **An alternative view: No-connectedness theorem.** The formalization just presented is based on the notion of closeness defined in terms of stimuli alone. This allows us to formulate the internal inconsistency of $\mathsf{Sup} \wedge \mathsf{Tol} \wedge \mathsf{Con}$ as $\mathsf{Sup} \wedge \mathsf{Con} \Longrightarrow \neg\mathsf{Tol}$, the 'no-tolerance' Theorem 3.5. There is, however, another way of approaching this inconsistency: to define vicinities as constant-response areas of the stimulus set S; we call these 'pi-vicinities' since we denote the stimulus-effect function by π. This makes the stimulus-effect function 'automatically' tolerant, and one sees subsequently that no two points in S are V-connected unless they map into one and the same stimulus effect: $\mathsf{Sup} \wedge \mathsf{Tol} \Longrightarrow \neg\mathsf{Con}$

Definition 3.7. Given a nonempty set S, an arbitrary set R, and a stimulus-effect function $\pi : S \to R$, the *pi-vicinity* of $x \in S$ is the set P_x of all $x' \in S$ such that $\pi(x') = \pi(x)$. The pair $\{S, \{P_x\}_{x \in S}\}$ is called the *pi-space associated to* π.

Lemma 3.8. *Any pi-space $\{S, \{P_x\}_{x \in S}\}$ uniquely corresponds to the V-space on S in which the only vicinity of $x \in S$ is P_x. The collection of the sets $\{P_x\}_{x \in S}$ is the only V-cover of S in this V-space.*

Proof. It is clear that $\{S, \{\mathcal{V}_x\}_{x \in S}\}$ with $\mathcal{V}_x = \{P_x\}$ satisfies Definition 3.1. $\square$

Lemma 3.9. *The pi-space $\{S, \{P_x\}_{x \in S}\}$ associated to $\pi : S \to R$ is uniquely determined by π, and π is tolerant in the corresponding V-space $\{S, \{\mathcal{V}_x = \{P_x\}\}_{x \in S}\}$.*

Proof. An immediate consequence of Definition 3.7, Lemma 3.8, and Definition 3.4. $\square$

We now easily obtain the 'no-connectedness' theorem:

Theorem 3.10. *Given the pi-space $\{S, \{P_x\}_{x \in S}\}$ associated to a stimulus-effect function $\pi : S \to R$, two elements $a, b \in S$ are V-connected in the corresponding V-space $\{S, \{\mathcal{V}_x = \{P_x\}\}_{x \in S}\}$ if and only if $\pi(a) = \pi(b)$.*

Proof. An immediate consequence of Definition 3.2 and the fact that either $P_x = P_y$ or $P_x \cap P_y = \varnothing$, for any $x, y \in S$. $\square$

4. Conclusion

We now summarize our main points, omitting (admittedly necessary) caveats and explanations.

(1) Sorites is not related to the issue of vagueness in human responses to stimuli. In fact, responses containing 'vague predicates' are always inconsistent, i.e., do not supervene on stimuli, and therefore do not allow one to formulate a sorites 'paradox'.

(2) The ‘paradox’ itself, when it can be formulated, is based on the false belief that responses of a ‘macroscopic’ system to two ‘microscopically different’ stimuli must be the same. In fact, whenever a stimulus space and stimulus effects can be defined in such a way that the effects supervene on stimuli and stimuli are connectable by ‘microscopic steps’, it is shown on a very high, pre-topological level of generality that the stimulus space must contain stimuli in any vicinity of which, however small, the stimulus effects vary.

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