Weak irregular principles

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28 February, 2012

Background

Applied computability theory looks at computable instances of various problems and studies the complexity of their solutions. Usual questions:

1. Is there always a computable solution?
2. Is there always a solution computable in $\emptyset'$?

Reverse mathematics seeks to calibrate the strength of (miniaturizations of) theorems according to the minimal sets of axioms needed to prove them. In practice, we use subsystems of second-order arithmetic:

1. first, we find a subsystem strong enough to prove the theorem;
2. then, we obtain sharpness, by showing that the theorem implies (reverses to) this subsystem, over some weak base theory.

There is a fruitful interaction between these two endeavors.

Background

RCA_0: basic axioms of arithmetic, together with induction for Σ^0_1 formulas, and comprehension for sets definable by both a Σ^0_1 and Π^0_1 formula.

WKL_0: RCA_0 plus every infinite binary tree has an infinite path.

ACA_0: RCA_0 plus comprehension for arithmetically-definable sets.

ATR_0: RCA_0 plus any arithmetically-defined functional $F : 2^{\mathbb{N}} \to 2^{\mathbb{N}}$ may be iterated along any countable well-ordering, starting with any set.

Π^1_1-CA_0: RCA_0 plus comprehension for Π^1_1-definable sets.

Background

Remarkably, most theorems fall into one of the “big five”.

Provable in RCA_0: Baire category theorem, intermediate value theorem, Urysohn’s lemma, Tietze extension theorem, soundness theorem.

Equivalent to WKL_0: every countable commutative ring has a prime ideal, Gödel’s compactness theorem, separable Hahn/Banach theorem, Heine/Borel theorem.

Equivalent to ACA_0: every countable commutative ring has a maximal ideal, every bounded sequence of real numbers has a least upper bound, Ascoli lemma, Bolzano/Weierstrass theorem, the range of every function exists, the Turing jump of every set exists.

$\vdots$

Background

$$\mathsf{Z}_2$$

$$\Downarrow$$

$$\vdots$$

$$\Downarrow$$

$$\Pi^1_1\text{-}\mathsf{CA}_0$$

$$\Downarrow$$

$$\mathsf{ATR}_0$$

$$\Downarrow$$

$$\mathsf{ACA}_0$$

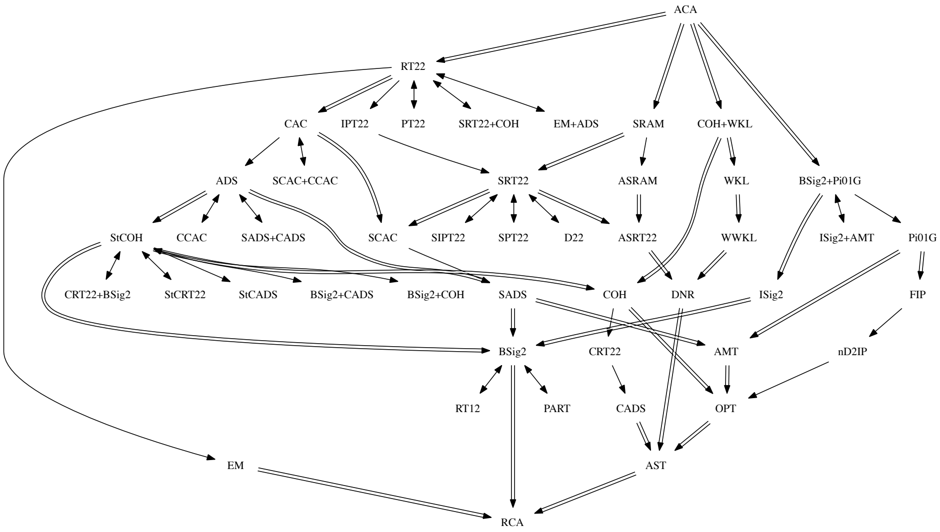
$$\Downarrow$$

$$\mathsf{WKL}_0$$

$$\Downarrow$$

$$\mathsf{RCA}_0$$

Background


ACA
RT22
CAC
IPT22
PT22
SRT22+COH
EM+ADS
SRAM
COH+WKL
ADS
SCAC+CCAC
SRT22
ASRAM
WKL
BSig2+Pi01G
StCOH
CCAC
SADS+CADS
SCAC
SIPT22
SPT22
D22
ASRT22
WWKL
ISig2+AMT
Pi01G
CRT22+BSig2
StCRT22
StCADS
BSig2+CADS
BSig2+COH
SADS
COH
DNR
ISig2
FIP
BSig2
CRT22
AMT
nD2IP
RT12
PART
CADS
OPT
EM
AST
RCA

Ramsey's theorem

Jockusch. Fix $n, k \geq 2$.

1 Every computable $f : [\omega]^n \to k$ has a Π^0_n infinite homogeneous set, but not necessarily a Δ^0_n (or Σ^0_n) one.

2 Every computable $f : [\omega]^n \to k$ has an infinite homogeneous set H with $H' \leq_T \emptyset^{(n)}$.

3 There exists a computable $f : [\omega]^n \to 2$ every infinite homogeneous set of which computes $\emptyset^{(n-2)}$.

Corollary. Over RCA_0, WKL_0 does not imply RT^2_2.

Corollary. For $n \geq 3$, RT^n_2 is equivalent to ACA_0 over RCA_0.

Ramsey's theorem for pairs

Seetapun. For any non-computable set C, every computable $f : [\omega]^2 \to 2$ has an infinite homogeneous set H that does not compute C.

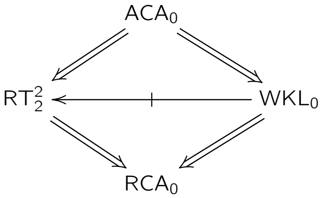
Cholak, Jockusch, and Slaman. Every computable $f : [\omega]^2 \to 2$ has a low_2 infinite homogeneous set H, i.e., $H'' \leq_T \emptyset''$.

Dzhafarov and Jockusch. Every computable $f : [\omega]^2 \to 2$ has a pair of infinite homogeneous sets H whose degrees form a minimal pair.

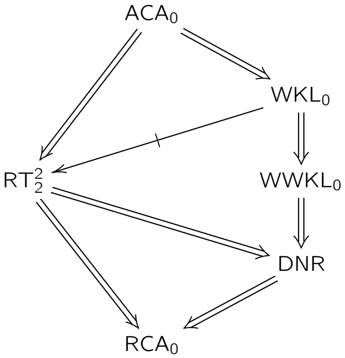
All proofs use some flavor of Mathias forcing.

Corollary. Over RCA_0, RT^2_2 does not imply ACA_0.

Ramsey's theorem for pairs



Ramsey's theorem for pairs



Ramsey's theorem for pairs

Liu. Every computable coloring $f : [\omega]^2 \to 2$ has an infinite homogeneous set not of PA degree.

Corollary. Over RCA_0, RT^2_2 does not imply WKL_0.

Dzhafarov and Shore. Every computable coloring $f : [\omega]^2 \to 2$ has a low_3 infinite homogeneous set H not of PA degree.

Question. Does every computable coloring $f : [\omega]^2 \to 2$ have an infinite low_2 homogeneous set not of PA degree?

Stability and cohesiveness

A coloring $f : [\omega]^2 \to 2$ is stable if for every x, $\lim_s f(\{x, s\})$ exists.

SRT^2_2: restriction of RT^2_2 to stable colorings.

COH: for every family of sets $\langle A_0, A_1, \ldots \rangle$ there is a set X such that for all i, either $X \subseteq^* A_i$ or $X \subseteq^* \overline{A}_i$.

If $A_0, A_1, \ldots$ contain all computable sets, X is r-cohesive; if $A_0, A_1, \ldots$ contain all c.e. sets, X is cohesive.

Cholak, Jockusch, and Slaman; Mileti; Jockusch and Lempp. Over RCA_0, RT^2_2 is equivalent to $\mathsf{SRT}^2_2 + \mathsf{COH}$.

Stability and cohesiveness

By the limit lemma, the computable content of SRT^2_2 is the same as that of the infinite subsets and co-subsets of Δ^0_2 sets. In particular, every computable stable coloring has a Δ^0_2 infinite homogeneous set.

Downey, Hirschfeldt, Lempp, and Solomon. There exists a computable stable coloring $f : [\omega]^2 \to 2$ with no low infinite homogeneous set.

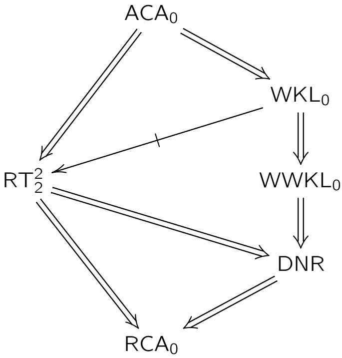
Corollary. Over RCA_0, WKL_0 does not imply SRT^2_2.

Hirschfeldt, Jockusch, Kjos-Hanssen, Lempp and Slaman. Over RCA_0, SRT^2_2 implies DNR, but COH does not.

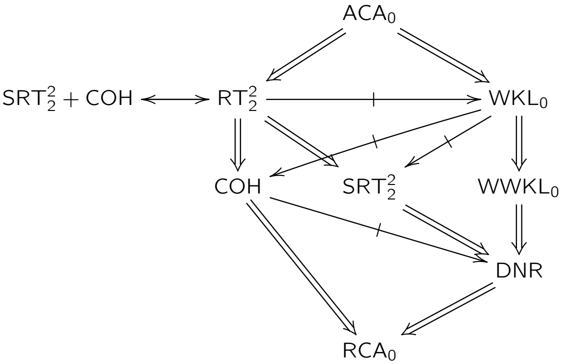
Chong, Slaman, Yang. Over RCA_0, SRT^2_2 does not imply RT^2_2.

The proof uses a highly customized, non-standard model of RCA_0.

Stability and cohesiveness



Stability and cohesiveness



Stability and cohesiveness

Question. Does SRT^2_2 imply COH in ω-models?

Another way to phrase the question:

Definition (Mileti). A degree $\mathbf{d}$ is s-Ramsey if every Δ^0_2 set has an infinite subset or cosubset in $\mathbf{d}$.

Question. Is every s-Ramsey degree a cohesive degree?

Mileti. The only Δ^0_2 degree is $\mathbf{0}'$. There is no low_2 s-Ramsey degree.

Measure-theoretic approach

Question. Does SRT^2_2 imply COH typically?

Definition.

1. A martingale is a function $M : 2^{<\omega} \to \mathbb{Q}^{\geq 0}$ such that for every $\sigma \in 2^{<\omega}$, $2M(\sigma) = M(\sigma 0) + M(\sigma 1)$.
2. M succeeds on a set X if $\limsup_n M(X \upharpoonright n) = \infty$. M succeeds on a class of sets $\mathcal{C}$ if it succeeds on every $X \in \mathcal{C}$.

Ville. A class of sets $\mathcal{C}$ has Lebesgue measure 0 if and only if there is a martingale that succeeds on it.

Measure-theoretic approach

Definition. A class $\mathcal{C}$ of Δ^0_2 sets is Δ^0_2 null if there is a $\emptyset'$-computable martingale M that succeeds on it.

Reasonable notion of nullity: additive, Δ^0_2 is not Δ^0_2 null, etc.

Hirschfeldt and Terwijn. The class of low sets is not Δ^0_2 null.

Definition (Dzhafarov). A degree $\mathbf{d}$ is almost s-Ramsey if the class of Δ^0_2 set having an infinite subset or cosubset in $\mathbf{d}$ is not Δ^0_2 null.

ASRT^2_2: non-Δ^0_2-null many Δ^0_2 sets have an infinite subset or cosubset.

Measure-theoretic approach

Dzhafarov.

1 The only Δ^0_2 almost s-Ramsey degree is $\mathbf{0}'$.

2 There exists an almost s-Ramsey degree $\mathbf{d} \leq \mathbf{0}''$ that is not s-Ramsey.

3 Over RCA_0, ASRT^2_2 is incomparable with COH. (And with WKL_0.)

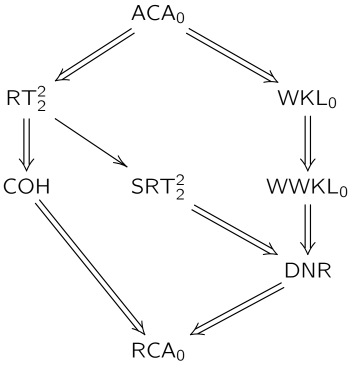
4 Over RCA_0, ASRT^2_2 is strictly stronger than DNR.

Proof idea for 2. Fix $A \in \Delta^0_2$ with no low infinite subset or cosset.

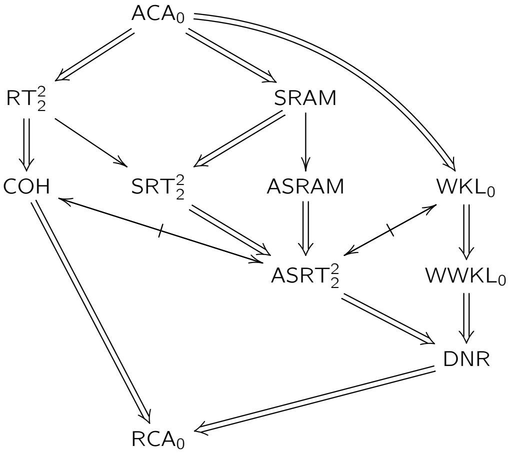
Let $M_0, M_1, \ldots$ list all $\emptyset'$-computable martingales, and for every i, fix L_i on which M_i does not succeed with $\bigoplus_{j \leq i} L_i$ low. Let $D^{[0]} = L_0$.

If Φ^D_0 is total, there must exist exist $n \notin A$ and a finite F such that $F^{[0]}$ agrees with $D^{[0]}$ and $\Phi^F_0(n) \downarrow = 1$. Let $D^{[1]}$ equal L_1 above $\varphi^F_0(n)$.

Measure-theoretic approach



Measure-theoretic approach



Variations of RT^2_2

Definition (Erdős and Rado). Fix a coloring $f : [\omega]^n \to 2$.

1. A p-homogeneous set for f is a sequence of infinite sets $\langle H_0, \ldots, H_{n-1}\rangle$ such that f is constant on all $\{x_0, \ldots, x_{n-1}\}$ with $x_i \in H_i$ for each $i < n$.
2. If f is merely constant on all such $\{x_0, \ldots, x_{n-1}\}$ with $x_0 < \cdots < x_{n-1}$, then $\langle H_0, \ldots, H_{n-1}\rangle$ is an increasing p-homogeneous set.

PT^n_k: every $f : [\omega]^n \to k$ has a p-homogeneous set.

IPT^n_k: every $f : [\omega]^n \to k$ has an increasing p-homogeneous set.

We can define stable versions, SPT^2_2 and SIPT^2_2, in the obvious way.

Variations of RT^2_2

Combinatorially, PT and RT^2_2 are rather different.

Define $f : [\omega]^2 \to 2$ by

$$f(x,y) = \begin{cases} 0 & x \equiv y \mod 2 \\ 1 & \text{else} \end{cases}$$

No infinite set is homogeneous for f with color 1, but $\langle \text{evens}, \text{odds} \rangle$ is a p-homogeneous set for f with color 1.

Variations of RT^2_2

It is easy to see that for all n, $\mathrm{RT}^n_2 \to \mathrm{PT}^n_2 \to \mathrm{IPT}^n_2$ over RCA_0.

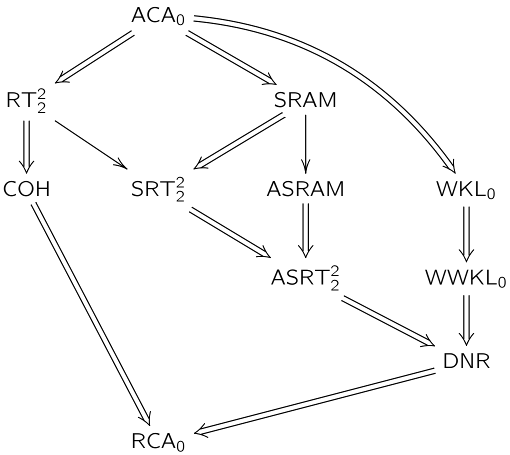
Dzhafarov and Hirst.

1. For $n \geq 3$, PT^n_k and IPT^n_k are equivalent to RT^n_k over RCA_0.
2. Over RCA_0, PT^2_2 is equivalent to RT^2_2.
3. Over $\mathrm{RCA}_0 + \mathrm{B}\boldsymbol{\Sigma}^0_2$, IPT^2_2 implies SRT^2_2.
4. Over $\mathrm{RCA}_0 + \mathrm{B}\boldsymbol{\Sigma}^0_2$, SPT^2_2 and SIPT^2_2 are equivalent to SRT^2_2.

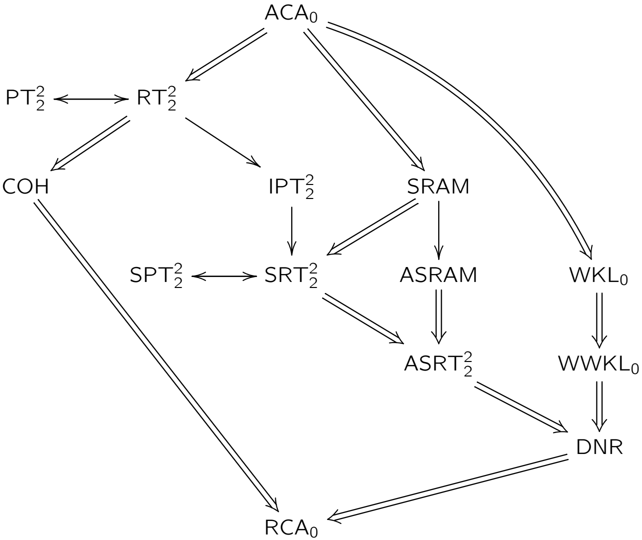
Proof outline of 2. To show: $\mathrm{PT}^2_2 \to \mathrm{RT}^2_2$. PT^2_2 implies SPT^2_2, hence SRT^2_2 over $B\boldsymbol{\Sigma}^0_2$. Thus, it suffices to show that PT^2_2 implies $\mathrm{COH} + \mathrm{B}\boldsymbol{\Sigma}^0_2$. Show PT^2_2 implies ADS, from which both COH and $B\boldsymbol{\Sigma}^0_2$ follow.

Chong, Lempp, and Yang. Parts 3 and 4 go through in $\mathrm{I}\boldsymbol{\Sigma}^0_1$.

Variations of RT^2_2



Variations of RT^2_2



Consequence of Ramsey's theorem

Hirschfeldt and Shore studied various consequences of RT^2_2:

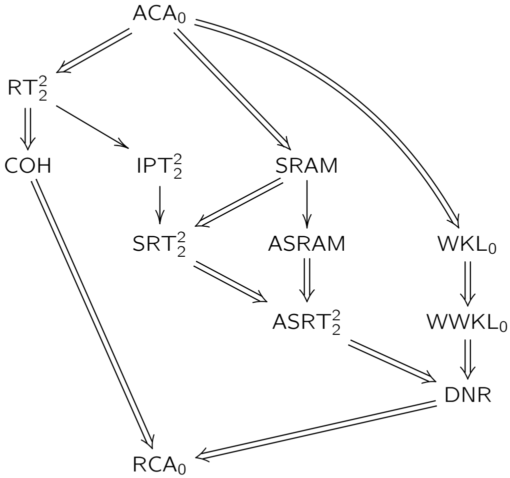
CAC: every partial ordering on ω has an infinite chain or an infinite antichain.

ADS: every linear ordering on ω has an infinite ascending sequence or an infinite descending sequence.

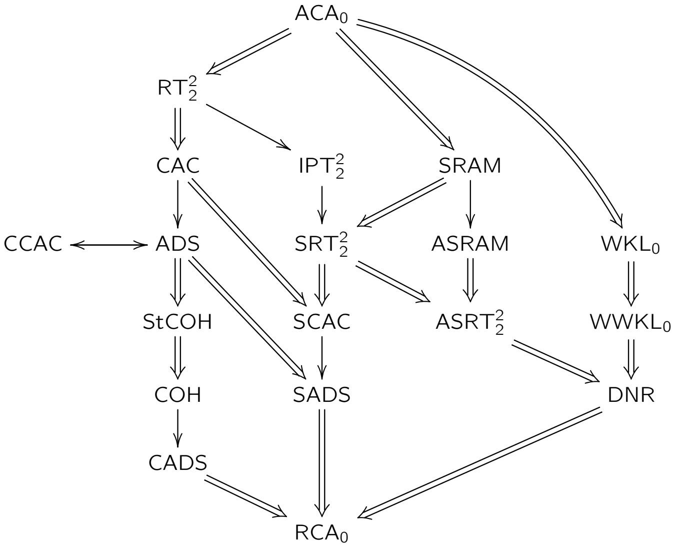
They also considered:

1. stable and cohesive versions of these principles: SCAC, SADS, CCAC, CADS, with $\mathsf{ADS} \leftrightarrow \mathsf{SADS} + \mathsf{CADS}$ and $\mathsf{CAC} \leftrightarrow \mathsf{SCAC} + \mathsf{CCAC}$.
2. a stronger version of COH, StCOH, equivalent to it under $\mathsf{B}\Sigma^0_2$.

Consequence of Ramsey's theorem



Consequence of Ramsey's theorem



Model-theoretic principles

A (countable, consistent) theory T is atomic if every T-consistent formula is provably implied by some atom (complete formula) of T.

A model of T is atomic if every type of T realized in it is principal.

Hirschfeldt, Shore, and Slaman studied the following:

AMT: every complete atomic theory has an atomic model.

OPT: for every theory T and every set S of partial types of T, there is a model of T that omits all the non-principal members of S.

Π^0_1G: for every uniformly Π^0_1 sequence $U_0, U_1, \ldots$ of dense subsets of $2^{<\omega}$, there exists a G that meets each U_i.

Model-theoretic principles

Hirschfeldt, Shore, and Slaman. The following hold in RCA_0:

1. $\Pi^0_1\mathsf{G} \to \mathsf{AMT}$, strictly.
2. $\mathsf{SADS} \to \mathsf{AMT} \to \mathsf{OPT}$, strictly.
3. $\mathsf{COH} \to \mathsf{OPT}$, strictly.
4. OPT is equivalent to the statement that for every X, there exists a set hyperimmune relative to X.

Some of the weakest principles that are not computably true.

Hirschfeldt, Shore, and Slaman also identified a principle that can claim to be the weakest such principle: AST is a weak form of OPT equivalent to the statement that for every X, there exists $Y \nleq_T X$.

Model-theoretic principles

Hirschfeldt, Shore, and Slaman.

1 AMT is Π^1_1 conservative over $\mathrm{B}\Sigma^0_2$.

2 AMT and $\Pi^0_1\mathrm{G}$ are r-Π^1_2 conservative over RCA_0; i.e., conservative for statements of the form

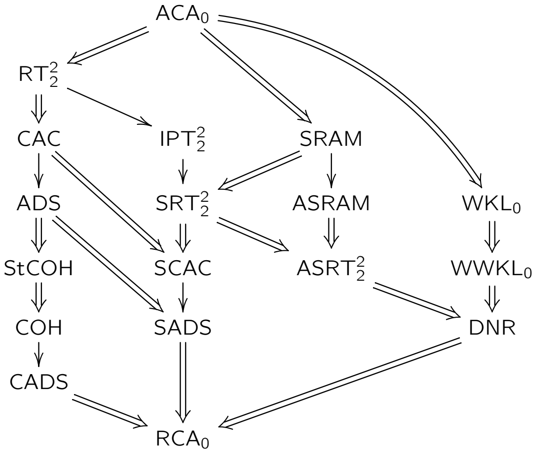
$$(\forall X)[\varphi(X) \to (\exists Y)\psi(X, Y)],$$

where φ is arithmetical and ψ is Σ^0_3.

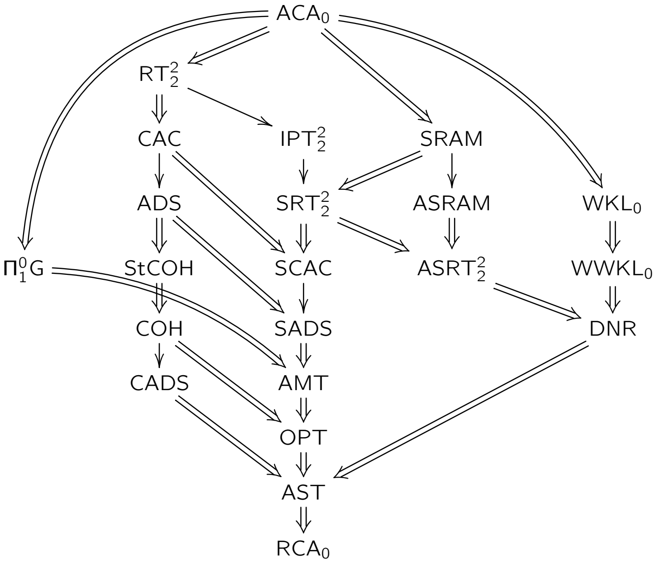
3 Over $\mathrm{RCA}_0 + \mathrm{B}\Sigma^0_2$, $\Pi^0_1\mathrm{G} \to \mathrm{I}\Sigma^0_2$. Over $\mathrm{RCA}_0 + I\Sigma^0_2$, $\mathrm{AMT} \to \Pi^0_1\mathrm{G}$.

r-Π^1_2 conservativity was first shown for COH. The same proof is used for all three principles, only the forcing notion differs: Cohen forcing for $\Pi^0_1\mathrm{G}$ and AMT, Mathias forcing for COH.

Model-theoretic principles



Model-theoretic principles



Equivalents of choice

Fix $n \geq 2$. A family A of sets has the

1. F intersection property if $\bigcap F \neq \emptyset$ for all finite $F \subseteq A$.
2. D_n intersection property if $\bigcap F = \emptyset$ for all $F \subseteq A$ of size n.
3. $\overline{D}_n$ intersection property if $\bigcap F \neq \emptyset$ for all $F \subseteq A$ of size n.

FIP: every non-trivial family has a maximal subfamily with the F intersection property.

D_nIP: every non-trivial family has a maximal subfamily with the D_n intersection property.

$\overline{D}_n$IP: every non-trivial family has a maximal subfamily with the $\overline{D}_n$ intersection property.

Equivalents of choice

Classically, FIP, D_nIP, and $\overline{D}_n$IP are equivalent to the axiom of choice.

Dzhafarov and Mummert.

1. For all n, D_nIP is equivalent to ACA_0 over RCA_0.
2. For all n, $F\mathsf{IP} \to D_{n+1}\mathsf{IP} \to D_n\mathsf{IP}$.
3. There is an ω-model of FIP consisting entirely of low sets.
4. FIP is r-Π^1_2 conservative over RCA_0.

Proof of 3. Force with conditions $(\sigma, s) \in \omega^{<\omega} \times \omega$ such that some number $\leq s$ belongs to $\bigcap_{i<|\sigma|} A_{\sigma(i)}$, and $(\tau, t) \leq (\sigma, s)$ if $\sigma \preceq \tau$. Let $(\sigma_0, s_0) = (\emptyset, 0)$. At even stages, force the jump. Given (σ_i, s_i) for i odd, choose the least $s \geq s_i$ large enough to bound an element of $A_i \cap \bigcap_{j<|\sigma|} A_{\sigma(j)}$ if non-empty, and let $(\sigma_{i+1}, s_{i+1}) = (\sigma_i \frown i, s)$.

Equivalents of choice

$\overline{D}_2$IP is computably false. The obvious computable strategy fails:

Given $A = \langle A_0, A_1, \ldots \rangle$ non-trivial, define a subfamily $B = \langle B_0, B_1, \ldots \rangle$ as follows. Having defined B_i for each $i < n$, search through A to find the first non-empty A_i not among the B_i but intersecting each of them, and let this be B_n.

B has the $\overline{D}_2$ intersection property, but it may fail to be maximal. It may be that A_0 intersects every set, but that it intersects $A_1, \ldots, A_n$ only after A_{n+1} does. Then B will equal $\langle A_1, A_2, \ldots \rangle$.

Can turn this problem into a proof that $\overline{D}_2$IP is not provable in RCA_0.

Equivalents of choice

Dzhafarov and Mummert. There exists a computable non-trivial $A = \langle A_0, A_1, \dots \rangle$ every maximal subfamily of which with the $\overline{D}_2$ intersection property has hyperimmune degree.

Proof sketch. We build A along with finitely-branching trees $T_0, T_1, \dots \subseteq \omega^{<\omega}$, and for every $J \in \omega^\omega$ a partial J-computable function f_J, so as to meet the following requirements:

- $\mathcal{Q}$: if $J \in \omega^\omega$ defines a maximal subfamily of A with the $\overline{D}_2$ intersection property then f_J is total;
- $\mathcal{R}_e$: if $J \in \omega^\omega$ defines a subfamily of A with the $\overline{D}_2$ intersection property, and if f_J is total and bounded by Φ_e, then $J \in [T_e]$;
- $\mathcal{S}_e$: no infinite path through T_e defines a maximal subfamily of A with the $\overline{D}_2$ intersection property.

Equivalents of choice

For each e, define sequence $P_e^0, P_e^1, \ldots \in A$ of e-prevention sets.

For some e, also define a missing set $M_e \in A$.

Goal: if J defines a maximal subfamily and f_J is total and bounded by Φ_e, then M_e intersects every $A_{J(i)}$, yet the P_e^m prevent M_e being in J.

Each P_e^m will be a prevention set for some $\sigma \in \omega^{<\omega}$, representing that any maximal subfamily containing all $A_{\sigma(i)}$ should also contain P_e^m.

At stage s:

1. we consider each $\sigma \in \omega^{<\omega}$ bounded by s, and define a new P_e^m for it;
2. for each P_e^m defined at a previous stage for some σ, look at each $\tau \succ \sigma$ bounded by s that only intersect M_e if σ does, and intersect P_e^m with every set enumerated by τ.

Equivalents of choice

Say σ e-extends τ if σ enumerates a P_e^m for some $\tau \preceq \rho \prec \sigma$.

For $J \in \omega^\omega$, f_J is defined as follows:

1. let $r_{-1} = 0$;
2. given r_{i-1} let r_i be least so that $J \upharpoonright r_i$ j-extends $J \upharpoonright r_{i-1}$ for each $j \leq i$;
3. let $f(0)$ be the least number that bounds $\bigcap_{j<r_i} A_{J(j)}$.

Equivalents of choice

Say σ *e*-extends τ if σ enumerates a P_e^m for some $\tau \preceq \rho \prec \sigma$.

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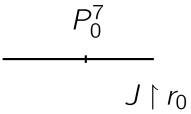
2 given r_{i-1} let r_i be least so that $J \restriction r_i$ j-extends $J \restriction r_{i-1}$ for each $j \leq i$;

3 let $f(0)$ be the least number that bounds $\bigcap_{j<r_i} A_{J(j)}$.



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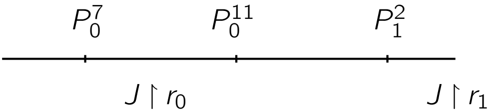

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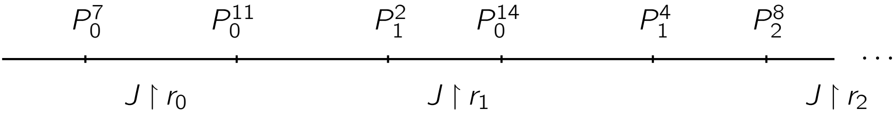


Equivalents of choice

Definition of T_e via approximations $T_e[0], T_e[1], \ldots$:

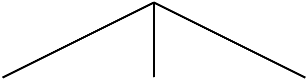
1. Let $T_e[0] = \emptyset$.
2. Given $T_e[s-1]$, suppose s is least such that $\Phi_{e,s}(i)\downarrow$.
3. Let $T_e[s]$ consist of all σ for which there exists a leaf τ of $T_e[s-1]$ such that σ j-extends τ for each $j \leq i$.

T_e is the intersection of the upward closure of the $T_e[s]$.

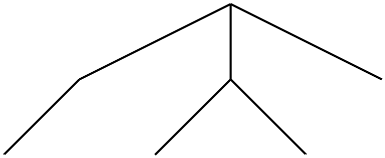
If $i = e$ above, define M_e.

If M_e is defined, and there is an r so that each leaf σ of $T_e[s]$ e-extends $\sigma \upharpoonright r$ with witness P_e^m disjoint from M_e and not enumerated by any string of length r, intersect $A_{\sigma(i)}$ with M_e for all $i < r$.

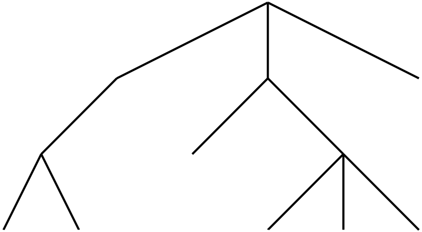
Equivalents of choice



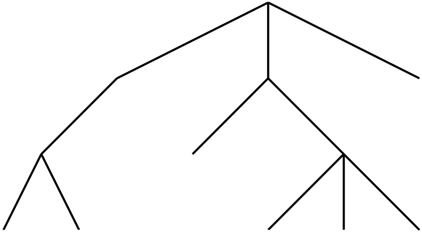
Equivalents of choice



Equivalents of choice

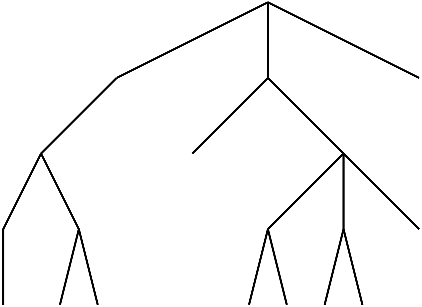


Equivalents of choice

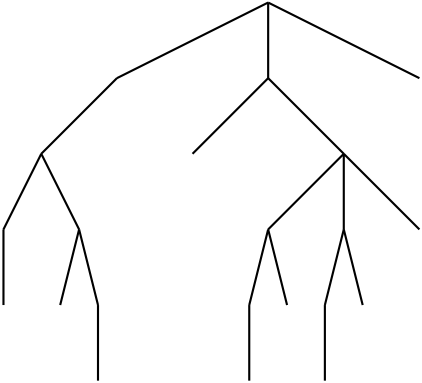


$\Phi_e(e)\downarrow$
define M_e

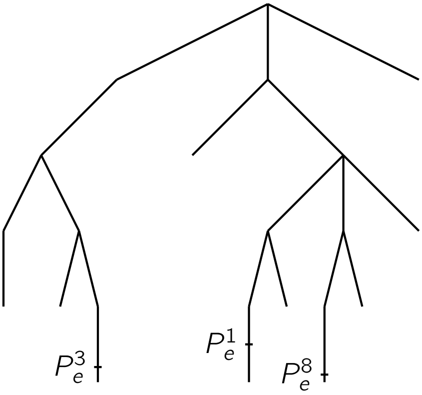
Equivalents of choice



Equivalents of choice

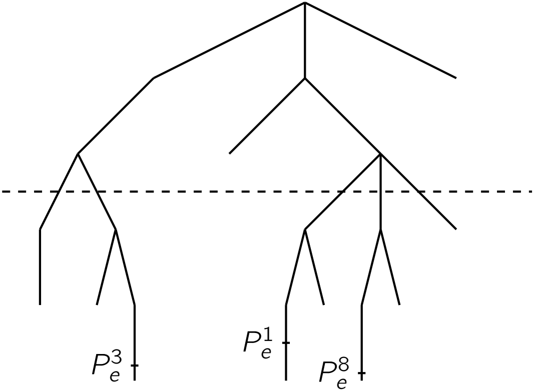


Equivalents of choice

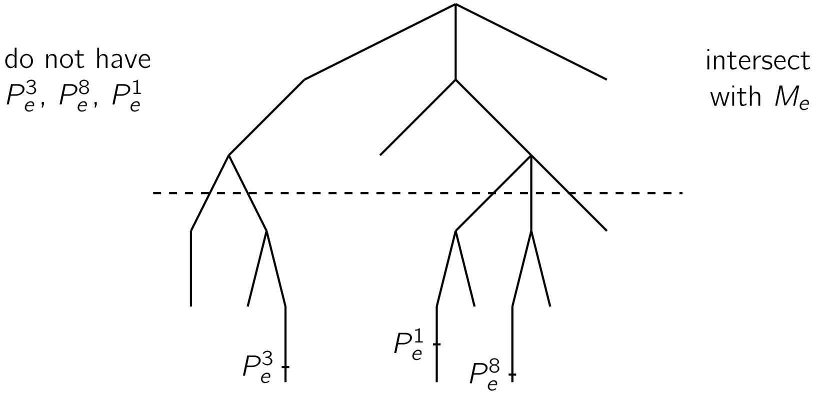


Equivalents of choice

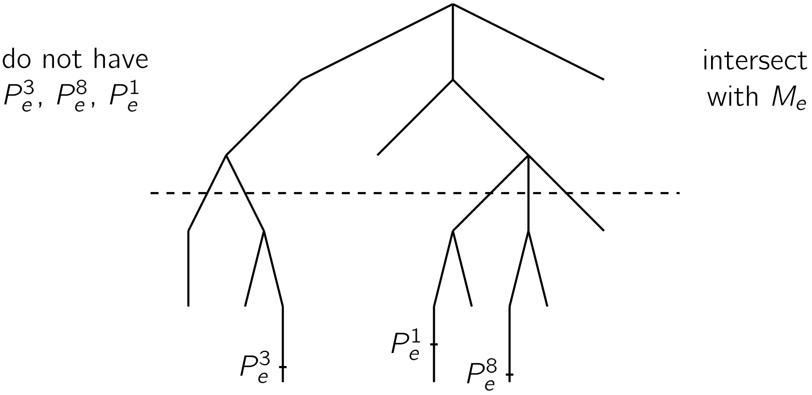
do not have
P_e^3, P_e^8, P_e^1



Equivalents of choice



Equivalents of choice



Ensures that M_e cannot be on by any path.

Equivalents of choice

Corollary. Over RCA_0, $\overline{D}_2\mathsf{IP}$ implies OPT.

Dzhafarov and Mummert.

1 Over RCA_0, $\Pi^0_1\mathsf{G}$ implies $F\mathsf{IP}$.

2 Every non-zero c.e. set can compute a maximal subfamily with the F intersection property of any computable non-trivial family of sets.

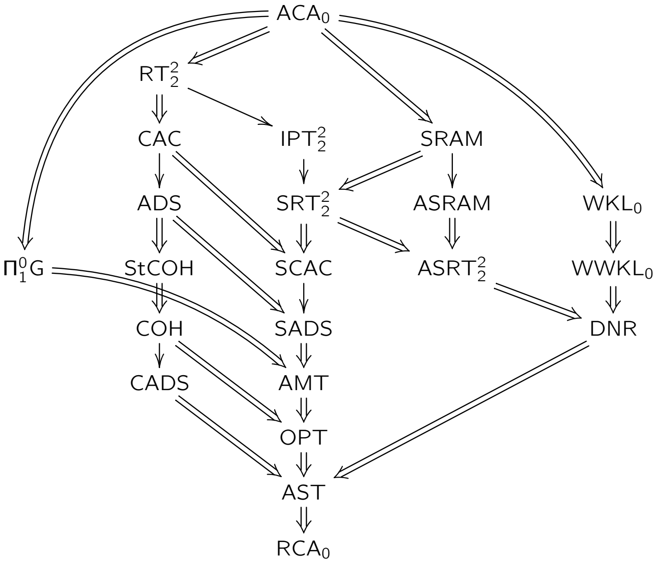
(Most other choice principles live above ATR_0, some even above Z_2.)

Csima, Hirschfeldt, Knight, and Soare. No low$_2$ Δ^0_2 set can compute an atomic model for every complete atomic decidable theory.

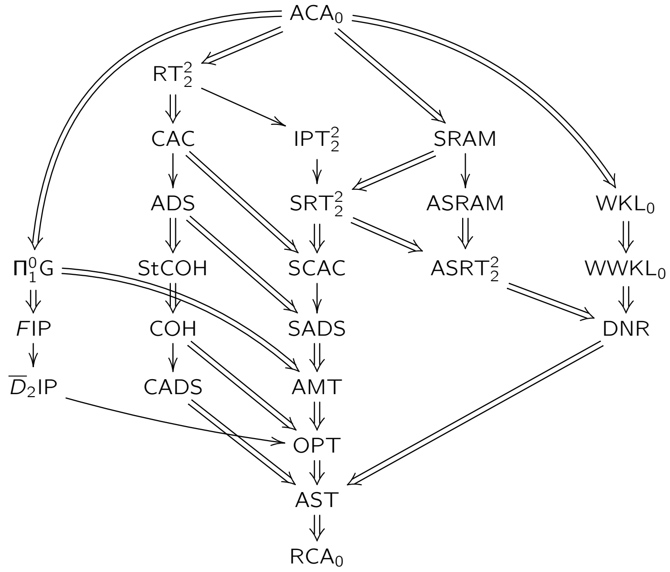
Corollary. There exists an ω-model of $\mathsf{RCA}_0 + F\mathsf{IP} + \neg\mathsf{AMT}$.

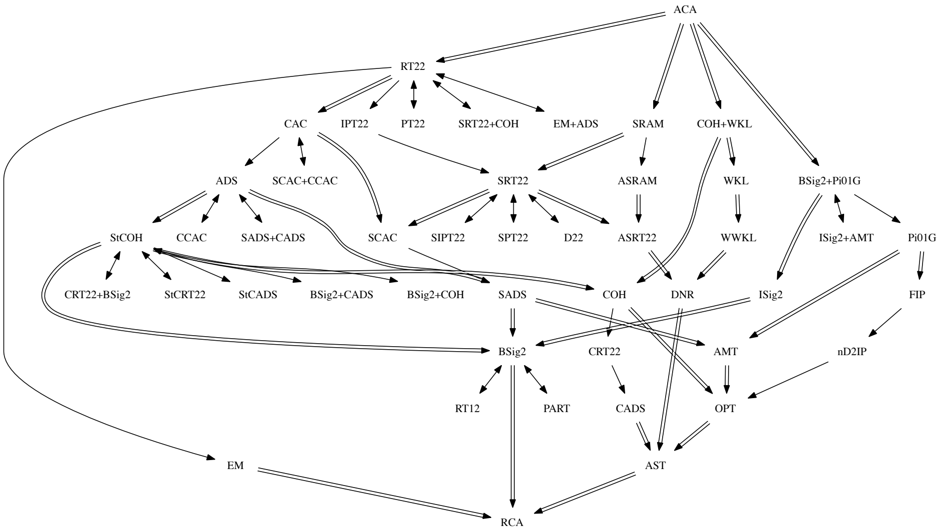
Question. Does OPT imply $\overline{D}_2\mathsf{IP}$? Does AMT imply $F\mathsf{IP}$?

Equivalents of choice



Equivalents of choice




ACA
RT22
CAC
IPT22
PT22
SRT22+COH
EM+ADS
SRAM
COH+WKL
ADS
SCAC+CCAC
SRT22
ASRAM
WKL
BSig2+Pi01G
StCOH
CCAC
SADS+CADS
SCAC
SIPT22
SPT22
D22
ASRT22
WWKL
ISig2+AMT
Pi01G
CRT22+BSig2
StCRT22
StCADS
BSig2+CADS
BSig2+COH
SADS
COH
DNR
ISig2
FIP
BSig2
CRT22
AMT
nD2IP
RT12
PART
CADS
OPT
EM
AST
RCA

Thank you for your attention.